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20/4/2020

Thm A subset of a topological space is open if and only if it is nbd. of each of its points.

Proof :- Let G be an open subset of a topological space. Then for every $x \in G$ we have $x \in G \subset G$ therefore G is a nbd. of each of its points. Conversely, let G be a nbd. of each of its points. If $G = \phi$, then it is open. If $G \neq \phi$, then to each $x \in G$ $\exists$ an open set G_x such that $x \in G_x \subset G$. It follows that

$G = \cup \{ G_x : x \in G \} \rightarrow G$ is an open set

Nbd. for Real line :- If $[a,b]$ is a closed interval on the real line, and $x \in [a,b]$ s.t. $a < x < b$ then for this system we can find nbd's also.

Ex. 1 - Is $[2,5]$ is a nbd. of 3

Proof :- Here $3 \in]2,5[\subset [2,5]$

(1) means $[2,5]$ is a nbd. of 3.

(2) Is $[2,5]$ is nbd. of 2.

Here $2 \notin]2,5[\subset [2,5]$

Here we can not find any open set containing 2 so $[2,5]$ is not a nbd of 2. Similarly $[2,5]$ is not a nbd. of 5.

(1) **Limit point** :- Let (X, τ) be a topological space and let $A \subset X$. A point $x \in X$ is called a limit point of A iff every nbd of x contains a point of A other than x

$$\Rightarrow (N_x - \{x\}) \cap A \neq \phi \quad \forall\ N_x$$

The set of all limit points of A is called derived set of A denoted by $D(A)$.

(2) **Adherent point** :- Let A be a subset of a topological space X and let $x \in X$. Then x is called an adherent point of A iff every nbd of x contains a point of A. The set of all adh. points of A is called $Adh(A)$

(3) **Closure** :- Let (X, τ) be a topological space & $A \subset X$. Then closure of A is denoted by

$$\bar{A} = cl(A) = \cap\{F : F \text{ is closed}, F \supset A\}$$

means $\bar{A}$ is always a closed set.
Here we can say that $\bar{A}$ is always a closed set. If A itself is a closed set then $A = \bar{A}$ otherwise $A \subset \bar{A}$

(4) **Properties of closure** -

(i) $\bar{\phi} = \phi$ (ii) $A \subset \bar{A}$ (iii) $A \subset B \Rightarrow \bar{A} \subset \bar{B}$

(iv) $\overline{A \cup B} = \bar{A} \cup \bar{B}$

④ **Interior of a set** :- Let (X, τ) be a topological space and $A \subset X$. A point $x \in A$ is said to be an interior point of A iff A is nbd. of x such that

$$x \in G \subset A$$

Set of all interior points of A is called interior of A and is denoted by $\mathring{A}$ or A^{i} or Int (A)

$$\boxed{\mathring{A} = \cup \{ G : G \text{ is open}, G \subset A \}}$$

$\mathring{A}$ is always an open set. If A is an open set then $A = A^{\circ}$, otherwise ~~$A = \mathring{A}$~~ $A^{\circ} \subset A$

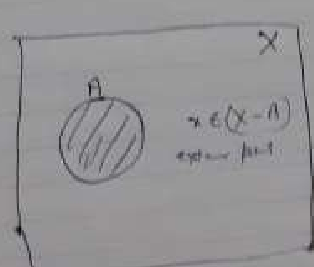
Properties :- (i) $X^{\circ} = X$, $\phi^{\circ} = \phi$

(ii) $A^{\circ} \subset A$ (iii) $A \subset B \Rightarrow A^{\circ} \subset B^{\circ}$

(iv) $(A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}$ (v) $A^{\circ} \cup B^{\circ} \subset (A \cap B)^{\circ}$

(vi) $A^{\circ\circ} = A$

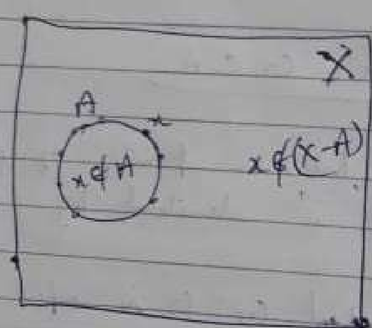
⑤ **Exterior points of a set** :- (X, τ) be a topological space



20/4/2020 Mon

(boundary point)

Frontier points of a set :- (X, τ) be a topl space. x is called a fron point iff it is neit int point nor an ext point of A. $Fr(A) =$ - -



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